Mini-Revision Sheet

1. What does it mean for a set A to be countable? Is the set of all rationals with prime denominator countable? Justify your answer.

2. What does it mean for the cardinality of a set X to be greater than or equal to the cardinality of a set Y? What is the relationship between the cardinality of $\mathbb{N}$ and the cardinality of the set of functions $f : \mathbb{N} \to \{0, 1\}$?

3. Exhibit a bijection that shows that $(0, 1]$ is a continuum.

4. True or false? If $(a_n)_{n\in\mathbb{N}}$ has a bounded subsequence, then $(a_n)_{n\in\mathbb{N}}$ has a convergent subsequence. Justify your answer.

5. Give two equivalent definitions of what it means for an element $a \in \mathbb{R}$ to be an accumulation point of the sequence $(a_n)_{n\in\mathbb{N}}$.

6. What is the $\liminf_{n\to\infty}$ of the sequence $(a_n)_{n\in\mathbb{N}}$ defined by $a_{2n} = 2n$, $a_{2n+1} = 1 + \frac{1}{n}$ for all $n \in \mathbb{N}$?

7. Let $(a_n)_{n\in\mathbb{N}}$ be defined by $a_n = (-1)^{n^2}$ for all $n \in \mathbb{N}$. Show that $(a_n)_{n\in\mathbb{N}}$ is not a Cauchy sequence, and deduce that $(a_n)_{n\in\mathbb{N}}$ does not converge.

8. Define what it means for a function to be uniformly continuous on its domain, and state Cantor's theorem on uniform continuity.

9. Is the sequence of functions $f_n : [0, \pi] \to \mathbb{R}$ defined by $f_n(x) = \sin^n(x)$ uniformly convergent? Justify your answer.